Unified analysis of pressure melting of ice around horizontal columns*

LIU Feng, CHEN Wenzhen**, MENG Bin and GONG Miao

(College of Power and Shipping, Naval University of Engineering, Wuhan 430033, China)

Accepted on September 5, 2006

Abstract The contact melting processes of ice, caused by pressure under the two-dimension axisymmetric horizontal columns, are generally studied. The unified mathematical expressions of the characteristic parameters for the pressure contact melting processes are obtained. Applying these expressions to the analysis of the pressure contact melting of ice around the horizontal cylinder, elliptical cylinder and flat plate, the related results in the published literatures are obtained, which prove the correctness and validity of the expressions. In addition, the expressions for the pressure contact melting of ice around the wedge-shaped object are also derived.

Keywords: **pressure melting, contact melting, horizontal columns, flat plate, wedge-shaped object**

The phenomena of contact melting exist widely in nature, which had attracted lots of scholars' attention. The contact melting caused by temperature difference have been studied broadly for various heat resources with different geometry shape[1—5]. In addition, some investigators made unified analysis about sphere, horizontal round tube, rectangle capsule and vertical round tube[6,7]. But there are some problems in the reports, namely either the force balance equation is singly listed as a complementary equation[6] without being analyzed and depicted in unification or the unified analysis can be only applied to the heat resources with given shape such as sphere, horizontal cylindrical tube, rectangular capsule and vertical cylindrical tube by employing the different factor[7]. So the results can not be applied to the contact melting under the heat resources with other shape. Bejan and Tyvand firstly put forward and analyzed the contact melting around an embedded horizontal cylinder driven by pressure in 1992[8,9], which is called as pressure contact melting. After that, some investigators studied the pressure contact melting of ice around horizontal cylinder, elliptical cylinder and sphere in succession[10—12], and some useful results were acquired. In the present work, the pressure contact melting under the axisymmetric horizontal column was analyzed in a general way, which took into consideration and avoided the above-mentioned problems of the unified analysis of temperature difference-driven contact melting. The unified expressions of some related characteristic parameters of pressure contact melting were derived. The expressions obtained were applied to the melting under the object with particular shape such as horizontal cylinder and elliptical cylinder as well as flat plate, and the results were proved to be correct by comparison with the related literatures'.

1 Theoretical analyses

Consider the pressure melting as shown in Fig. 1, the infinite long horizontal column melts down in the ice with velocity U under the force F. The curve equation of horizontal column section is $y = f(x)$. (x_0, y_0) is the right end point of the section. The temperature of ice and horizontal column are both supposed to be $T_0 = 0℃$ at initial time. For simplicity, it is assumed that T_0 approximately equals to the temperature of triple-point of water. The melting point of ice drops to $T_m (< T_0)$ because of the increase of pressure p and the ice changes into water. According to the Clausius-Clapeyron equation the relationship between melting point of ice T_m and pressure p is

$$dp/dT_m = -A, \qquad (1)$$

where $A = L/[T_0(v_s - v_1)]$ and L is melting latent heat. v_s and v_1 are the specific volume of ice and water, respectively. Around the triple-point of water, $A \cong 13.6$ MPa/K. Integrating Eq. (1) yields

$$T_0 - T_m = (p - p_0)/A. \qquad (2)$$

It is assumed that (a) the boundary layer thick-

* Supported by National Natural Science Foundation of China (Grant No. 50376074)

** To whom correspondence should be addressed. E-mail: cwz2@21cn.com

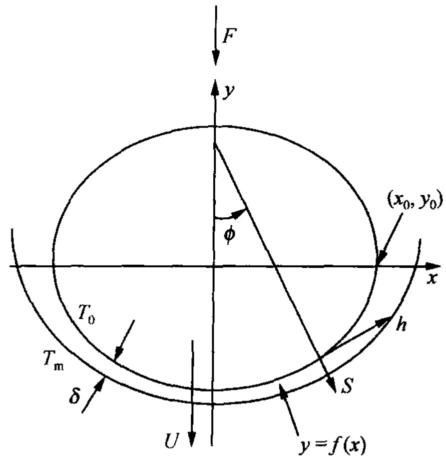

Fig. 1. Physical model and coordinates.

ness is much smaller than the radial size of horizontal column, and heat transport by the convective flow is negligible as compared with that by heat conduction in the quasi steady liquid layer; (b) neglecting the effect of the inertia force, the momentum equation in the boundary layer is caused mainly by the viscous forces and $\partial^2/\partial h^2 \ll \partial^2/\partial s^2$. Then the governing equations and boundary conditions can be written as follows, respectively

$$\mu \frac{\partial^2 u}{\partial s^2} = \frac{\mathrm{d}p}{\mathrm{d}h}, \tag{3}$$

$$\lambda_1 \frac{T_0 - T_m}{\delta} = \rho U L \cos\phi, \tag{4}$$

$$\int_0^\delta u \,\mathrm{d}s = \int_0^h U \cos\phi \,\mathrm{d}h, \tag{5}$$

$$\left.\begin{array}{l} s = 0,\ u = 0,\ T = T_0 \\ s = \delta,\ u = 0,\ T = T_m \end{array}\right\}, \tag{6}$$

where u, μ, λ_1 and ρ are velocity of liquid along h direction inside boundary layer, dynamic viscosity, thermal conductivity and density of liquid, respectively. The expression of normal angle ϕ is

$$\cos\phi = \left(\frac{1}{1 + \tan^2\phi}\right)^{\frac{1}{2}} = \left(\frac{1}{1 + (f'(x))^2}\right)^{\frac{1}{2}}. \tag{7}$$

Combing Eqs. (3) and (6), we can get

$$u = \frac{1}{2\mu} \frac{\mathrm{d}p}{\mathrm{d}h} s(s - \delta). \tag{8}$$

Substituting Eq. (8) into Eq. (5) yields

$$\frac{\mathrm{d}p}{\mathrm{d}h} = -12 \frac{\mu x U}{\delta^3}. \tag{9}$$

Substituting Eq. (2) into Eq. (4) yields

$$\delta = \frac{\lambda_1 (p - p_0)}{\rho L U A \cos\phi}. \tag{10}$$

Substituting Eqs. (10) and (7) into Eq. (9) results in

$$p - p_0 = U\left(\frac{48\mu\rho^3 L^3 A^3}{\lambda_1^3}\right)^{1/4} \left(\int_x^{x_0} \frac{x}{1 + (f'(x))^2} \mathrm{d}x\right)^{1/4}. \tag{11}$$

The pressure on unit length of horizontal column is

$$F' = 2\int_0^{x_0} (p - p_0) \mathrm{d}x = 2U\left(\frac{48\mu\rho^3 L^3 A^3}{\lambda_1^3}\right)^{1/4} \cdot \int_0^{x_0} \left(\int_x^{x_0} \frac{x}{1 + (f'(x))^2} \mathrm{d}x\right)^{1/4} \mathrm{d}x. \tag{12}$$

The average pressure on the horizontal column surface is

$$F'' = \frac{F'}{2x_0}. \tag{13}$$

Substituting Eq. (12) into Eq. (13) results in

$$U = \frac{x_0 F''}{\left(\frac{48\mu\rho^3 L^3 A^3}{\lambda_1^3}\right)^{1/4} \int_0^{x_0} \left(\int_x^{x_0} \frac{x}{1 + (f'(x))^2} \mathrm{d}x\right)^{1/4} \mathrm{d}x}. \tag{14}$$

Substituting Eqs. (7) and (11) into Eq. (10), we can get the liquid film thickness

$$\delta = \left(\frac{48\mu\lambda_1}{\rho L A}\right)^{1/4} (1 + (f'(x))^2)^{1/2} \cdot \left(\int_x^{x_0} \frac{x}{1 + (f'(x))^2} \mathrm{d}x\right)^{1/4}. \tag{15}$$

Eqs. (11)—(15) are the unified mathematical expressions of the characteristic parameters for the pressure melting of ice around axis-symmetric horizontal columns.

2 Results and discussion

2.1 The pressure melting of ice around the horizontal cylinder

When the horizontal column is a cylinder with radius R, the section curve equation and right end point of the cylinder are given as follows, respectively,

$$\left.\begin{array}{l} f(x) = -(R^2 - x^2)^{1/2} \\ x_0 = R \end{array}\right\}. \tag{16}$$

Then, we can derive

$$\left(\int_x^{x_0} \frac{x}{1 + (f'(x))^2} \mathrm{d}x\right) = \frac{1}{R^2}\left(\frac{R^2}{2} - \frac{x^2}{2}\right)^2, \tag{17}$$

$$\int_0^{x_0} \left(\int_x^{x_0} \frac{x}{1 + (f'(x))^2} \mathrm{d}x\right)^{1/4} \mathrm{d}x = 0.5^{2.5} \pi R^{1.5}. \tag{18}$$

Substituting Eqs. (17) and (18) into Eqs. (11)—(15), respectively, results in

$$p - p_0 = U\left(\frac{12\mu R^2 L^3 \rho^3 A^3}{\lambda_1^3}\right)^{1/4}\left(1 - \left(\frac{x}{R}\right)^2\right)^{1/2}, \quad (19)$$

$$F' = \frac{\pi}{2} U R^{3/2}\left(\frac{(12\mu)^{1/3}\rho L A}{\lambda_1}\right)^{3/4}, \quad (20)$$

$$F'' = \frac{F'}{2R} \approx 1.462 U R^{1/2}\left(\frac{\mu^{1/3}\rho L A}{\lambda_1}\right)^{3/4}, \quad (21)$$

$$U \approx F'' \Big/ \left(1.462 R^{1/2}\left(\frac{\mu^{1/3}\rho L A}{\lambda_1}\right)^{3/4}\right), \quad (22)$$

$$\delta = \left(\frac{12\mu\lambda_1}{\rho L A}\right)^{1/4} R^{1/2}. \quad (23)$$

Eqs. (19)—(23) are the same with the results obtained by literature[10].

2.2 The pressure melting of ice around the elliptical cylinder

When the horizontal column is a horizontal elliptical cylinder with long and short radius b, a, the section curve equation and right end point of the elliptical cylinder are given as follows, respectively,

$$\left.\begin{aligned} f(x) &= -b\left(1 - \frac{x^2}{a^2}\right)^{1/2} \\ x_0 &= a \end{aligned}\right\}. \quad (24)$$

Then, we can derive

$$\int_x^{x_0} \frac{x}{1 + (f'(x))^2} dx = \frac{a^2}{2} \frac{(1 - J^2)(1 - (x/a)^2) + J^2 \ln \dfrac{J^2}{1 - (x/a)^2 + J^2 (x/a)^2}}{(1 - J^2)^2}, \quad (25)$$

$$\int_0^{x_0}\left(\int_x^{x_0} \frac{x}{1 + (f'(x))^2} dx\right)^{1/4} dx = 2^{-1/4} a^{3/2} \int_0^1 \left[\frac{(1 - J^2)(1 - z^2) + J^2 \ln \dfrac{J^2}{1 - z^2 + J^2 z^2}}{(1 - J^2)^2}\right]^{1/4} dz, \quad (26)$$

where $J = b/a$. Substituting Eqs. (25) and (26) into Eqs. (11)—(15), respectively, results in

$$p - p_0 = U\left(\frac{24\mu a^2 \rho^3 L^3 A^3}{\lambda_1^3}\right)^{1/4} \times \left[\frac{(1 - J^2)(1 - (x/a)^2) + J^2 \ln \dfrac{J^2}{1 - (x/a)^2 + J^2 (x/a)^2}}{(1 - J^2)^2}\right]^{1/4}, \quad (27)$$

$$F' = 2\int_0^{x_0} (p - p_0) dx = f_1(J) 2a^{3/2} U\left(\frac{24\mu\rho^3 L^3 A^3}{\lambda_1^3}\right)^{1/4}, \quad (28)$$

$$U = F'' \Big/ \left(f_1(J) a^{1/2}\left(\frac{24\mu\rho^3 L^3 A^3}{\lambda_1^3}\right)^{1/4}\right), \quad (29)$$

$$\delta = \left(\frac{24 a^2 \mu\lambda_1}{\rho L A}\right)^{1/4}\left(1 + \frac{J^2 (x/a)^2}{1 - (x/a)^2}\right)^{1/2} \times \left[\frac{(1 - J^2)(1 - (x/a)^2) + J^2 \ln \dfrac{J^2}{1 - (x/a)^2 + J^2 (x/a)^2}}{(1 - J^2)^2}\right]^{1/4}, \quad (30)$$

where

$$f_1(J) = \int_0^1 \left[\frac{(1 - J^2)(1 - z^2) + J^2 \ln \dfrac{J^2}{1 - z^2 + J^2 z^2}}{(1 - J^2)^2}\right]^{1/4} dz. \quad (31)$$

Eqs. (27)—(31) are the primary results obtained by literature[12]. When $a = b = R$, namely $J = 1$, the elliptical cylinder changes into a cylinder. Then from Eqs. (27)—(31) we can also obtain Eqs. (19)—(23).

2.3 The pressure melting of ice under the horizontal flat plate

When the horizontal column is an infinite long flat plate, the section curve equation and right end point of the flat plate are given as follows, respectively,

$$\left.\begin{aligned} f(x) &= C \\ x_0 &= a \end{aligned}\right\}, \quad (32)$$

where C is a random constant, and a is half of the width of the flat plate. Substituting Eq. (32) into Eqs. (11)—(15), respectively, results in

$$p - p_0 = U\left(\frac{24\mu\rho^3 L^3 A^3}{\lambda_1^3}(a^2 - x^2)\right)^{1/4}, \quad (33)$$

$$F' = 2\int_0^{x_0} (p - p_0) dx = 2a^{3/2} U\left(\frac{24\mu\rho^3 L^3 A^3}{\lambda_1^3}\right)^{1/4} \int_0^1 (1 - z^2)^{1/4} dz. \quad (34)$$

Transforming the function $(1-z^2)^{1/4}$ in Eq. (34) into exponential series and substituting it with a vanishing error by a fifth-order polynominal, we can derive

$$F' \approx 2a^{3/2}U\left(\frac{24\mu\rho^3L^3A^3}{\lambda_1^3}\right)^{1/4} \times \int_0^1\left(1-\frac{z^2}{4}-\frac{9z^4}{96}-\frac{315z^6}{5760}-\frac{24255z^8}{645120}\right)dz \approx 3.9218a^{3/2}U\left(\frac{\mu\rho^3L^3A^3}{\lambda_1^3}\right)^{1/4}, \quad (35)$$

$$F'' \approx 1.9609a^{1/2}U\left(\frac{\mu\rho^3L^3A^3}{\lambda_1^3}\right)^{1/4} = 1.3866UW^{1/2}\left(\frac{\mu\rho^3L^3A^3}{\lambda_1^3}\right)^{1/4}, \quad (36)$$

$$\delta = \left(\frac{24\mu\lambda_1}{\rho LA}\right)^{1/4}(a^2-x^2)^{1/4}. \quad (37)$$

Comparing Eqs. (36) and (37) with Bejan's[8] Eqs. (16) and (15), we can find that the expressions of boundary layer thickness are the same. The coefficient in Eq. (15) of Ref. [8] is 1.368, and here it is 1.3866, which means the relationship between average pressure and melting velocity is also the same approximately (in Eq. (36). $W=2a$).

2.4 The pressure melting of ice around the horizontal wedge-shaped object

When the horizontal column is a wedge-shaped object, we can get

$$\left.\begin{aligned} f(x) &= -aC + Cx \\ x_0 &= a \end{aligned}\right\}, \quad (38)$$

where C is a random constant, a is half of the width of the wedge-shaped object. Substituting Eq. (38) into Eqs. (11)—(15), respectively, results in

$$p - p_0 = U\left(\frac{24\mu\rho^3L^3A^3}{\lambda_1^3(1+C^2)}(a^2-x^2)\right)^{1/4}, \quad (39)$$

$$F'' \approx 1.9609a^{1/2}U\left(\frac{\mu\rho^3L^3A^3}{\lambda_1^3(1+C^2)}\right)^{1/4}, \quad (40)$$

$$\delta = \left(\frac{24\mu\lambda_1(1+C^2)}{\rho LA}\right)^{1/4}(a^2-x^2)^{1/4}. \quad (41)$$

Eqs. (39)—(41) are the expressions of pressure melting of ice around a horizontal wedge-shaped object. From Eqs. (39) to (41), it can be seen that the sharper the wedge-shaped object is (C is bigger), the bigger the melting velocity and the thicker the boundary layer are. When $C=0$, the wedge-shaped object changes into a flat plate, then we can derive the Eqs. (36) and (37) from Eqs. (40) and (41), respectively.

3 Conclusions

In this paper, the general physical model is proposed and the unified mathematical expressions of the characteristic parameters for the pressure melting processes of ice around the axis-symmetric horizontal column are derived, which is proved to be correct by being applied to the analysis of the pressure melting around the horizontal cylinder, elliptical cylinder and flat plate. In addition, the mathematic expressions for the pressure contact melting of ice around the wedge-shaped object are also derived. The results obtained in this paper offer a quicker and simpler means for the unified analysis and description of the pressure melting around a kind of infinite long horizontal column.

References

1 Emerman S. H., Turcotte D. L. Stokes problem with melting. Int. J. Heat Mass Transfer, 1983, 26(11): 1625—1630.
2 Moallemi M. K., Viskanta R. Experiments on flow induced by melting around a migrating heat source. J. Fluid Mech., 1985, 157: 35—51.
3 Roy S. K., Sengupta S. A generalized model for gravity-assisted melting in enclosures. J. Heat Transfer, 1990, 112: 804—808.
4 Vargas J. V. C., Bejan A., Dobrovicescu A. The melting of an ice shell on a heated horizontal cylinder. J. Heat Transfer, 1994, 116: 702—708.
5 Chen W. Z., Yang Q. S., Dai M. Q., et al. Analytical solution of the heat transfer process during contact melting of phase change material inside a horizontal elliptical tube. Int. J. Energy Res., 1998, 22(2): 131—140.
6 Oka M. and Carey V. P. A unified treatment of the direct contact melting processes in several geometric cases. Int. Comm. Heat Mass Transfer, 1996, 23: 187—202.
7 Hu Y. J., Huang S. Y., Shi M. H. A generalized analysis on close-contact melting processes in two-dimensional axisymmetric geometries. Acta Energiae Solaris Sinica (in Chinese), 1999, 20 (3): 324—329.
8 Bejan A., Tyvand P. A. The pressure melting of ice under a body with flat base. J. Heat Transfer, 1992, 114(2): 529—531.
9 Tyvand P. A., Bejan A. The pressure melting of ice due to an embedded cylinder. J. Heat Transfer, 1992, 114(2): 532—535.
10 Chen W Z, Yang Q S. The pressure melting of ice around the sphere and cylinder. Proceeding of the 11th International Heat Transfer Conference. Korea, 1998, 181—186.
11 Chen W. Z., Li H. F., Gao M. The pressure melting of ice around the elliptical cylinder with finite contact length. Journal of engineering thermophysics, 2005, 26(6): 994—996 (in Chinese).
12 Chen W. Z., Li H. F., Gao M., et al. The pressure melting of ice around a horizontal elliptical cylinder. Heat and Mass Transfer, 2005, 42: 138—143.